

Mathematical Measures of Compactness for Legislative Districts

There is no official definition of a compact district, and many people instinctively rely on the ‘I know it when I see it’ test. In general, a compact district has a regular shape, with all constituents relatively close to one another (1).

Mathematical Measures of Compactness

At least 30 measures of district or district plan compactness have been proposed by scholars in disciplines ranging from economics to law to mathematics. Each test places priority on a certain characteristic of the district (1).

Many tests focus on the districts perimeter and measure boundary contortion. In general, these tests consider the length of district boundaries or the relationship between a district’s perimeter and its area. Under these methods, a district with smooth boundaries will be more compact than a district with more jagged or “squiggly” boundaries. A commonly referenced measure of boundary contortion is the “Polsby- Popper” test (1; 2; 3; 4).

Other methods focus on dispersion, or how far the district spreads out from a central area. Under these measures, a district with many “fingers” or “tendrils” would be less compact. Commonly referenced measures of dispersion include the “Reock” test (1; 3; 2; 5).

Finally, some measures focus on population dispersion within a district. A district with a population center close to its geographic center would be more compact (1). Examples of these types of tests include the “Population Circle” and “Convex Hull” (3). Recent work has also focused on “paths” or connections within a district (3).

Use of Mathematical Measures of Compactness

Compactness must be considered alongside other important principles such as equal population, preserving political boundaries, or representing a community of interest (1). A gerrymandered district can appear compact, and fair districts drawn to achieve other principles may have irregular shapes (6). Tests of compactness are one piece in the overall evaluation of a plan.

Each of the measures of compactness prioritizes a certain characteristic, and different measures can give different results for an individual district (3). Scholars caution against applying mathematical measures to a single district or setting a threshold number that determines when a district can be considered compact. Rather, measures are best used to compare plans or identify a district within a plan that is less compact than others. In addition, multiple tests should be used to evaluate plans (3; 7; 8).

Use of Compactness Measures in Other States

State	Specifications for Measurement of Compactness
Arizona	The 2001 Commission specified that compactness be measured using the Polsby-Popper Test. The 2010/11 Commission did not specify a test of compactness (9).
Colorado	Constitution specifies that “each district be as compact as possible and the aggregate linear distance of all district boundaries be as short as possible (10).”
Michigan	Statute requires that “compactness be determined by circumscribing each district within a circle of minimum radius and measuring the area, not part of the Great Lakes and not part of another state, inside the circle but not inside the district (11)”.
Montana	Code states that “The districts must be compact, meaning that the compactness of a district is greatest when the length of the district and the width of the district are equal. A district may not have an average length greater than three times the average width unless necessary to comply with the Voting Rights Act (12).”
Iowa	Iowa code requires reasonably compact districts to the extent possible when first taking into account equal

State	Specifications for Measurement of Compactness
	<p>population, the VRA, and contiguousness. Reasonably compact districts are “square, rectangular, or hexagonal and not irregularly shaped” as allowed by natural or political boundaries (13).</p> <p>Mathematical measures of compactness are to be used when necessary to compare districts or plans. These include an aggregate measure of length-width compactness for all districts in a plan, and an aggregate measure of perimeter length for all districts in a plan.</p>
Minnesota	A 2011 Special Judicial Redistricting Panel issued an order requiring that submissions of redistricting plans include a report stating the results of Reock, Schwartzberg, Perimeter, Polsby-Popper, Length-Width, Population Polygon, Population Circle, and Ehrenberg measures of compactness (14).

References

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